Computer Graphics

Lecture 14

Two dimensional Geometric transformation Continued

Scaling: A scaling transformation alters the size of an object. This operation can be carried out for polygons by multiplying the coordinate values (x, y) of each vertex by scaling factors s_x and s_y to produce the transformed coordinates (x', y'):

$$x' = x \cdot s_x , \quad y' = y \cdot s_y \qquad (5\text{-}10)$$

Scaling factor s_x scales objects in the x direction, while s_y scales in the y direction. The transformation equations 5-10 can also be written in the matrix form:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \qquad (5\text{-}11)$$

Or

$$P' = S \cdot P \qquad (5\text{-}12)$$

where S is the 2 by 2 scaling matrix in Eq. 5-11.

Any positive numeric values can be assigned to the scaling factors s_x and s_y. Values less than 1 reduce the size of objects; values greater than 1 produce an enlargement. Specifying a value of 1 for both s_x and s_y leaves the size of objects unchanged. When s, and s, are assigned the same value, a **uniform scaling** is produced that maintains relative object proportions. Un-equal values for s_x and s_y result in a differential scaling that is often used in design applications, whew pictures are constructed from a few basic shapes that can be adjusted by scaling and positioning transformations (Fig. 5-6).

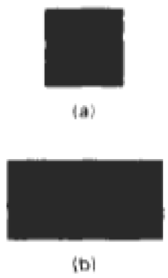

Figure 5-6
Turning a square (a) into a rectangle (b) with scaling factors $s_x = 2$ and $s_y = 1$.

Objects transformed with Eq. 5-11 are both scaled and repositioned. Scaling factors with values less than 1 move objects closer to the coordinate origin, while values greater than 1 move coordinate positions farther from the origin. Figure 5-7 illustrates scaling a line by assigning the value 0.5 to both s_x and s_y in Eq. 5-11. Both the line length and the distance from the origin are reduced by a factor of 1/2.

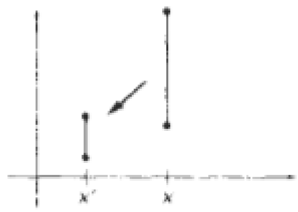

Figure 5-7
A line scaled with Eq 5-12 using $s_x = s_y = 0.5$ is reduced in size and moved closer to the coordinate origin.

We can control the location of a scaled object by choosing a position, called the fixed point that is to remain unchanged after the scaling transformation. Coordinates for the fixed point (x_f, y_f) can be chosen as one of the vertices, the object centroid, or any other position (Fig. 5-8). A polygon is then scaled relative to the fixed point by scaling the distance from each vertex to the fixed point. For a vertex with coordinates (x, y) the scaled coordinates(x ' , y ') are calculated as

$$x' = x_f + (x - x_f)s_x, \qquad y' = y_f + (y - y_f)s_y \tag{5-13}$$

We can rewrite these scaling transformations to separate. The multiplicative and additive terms:

$$\begin{aligned} x' &= x \cdot s_x + x_f(1 - s_x) \\ y' &= y \cdot s_y + y_f(1 - s_y) \end{aligned} \tag{5-14}$$

where the additive terms $x_f\,(1- s_x)$ and $y_f\,(1 - s_y)$ are constant for all points in the object.

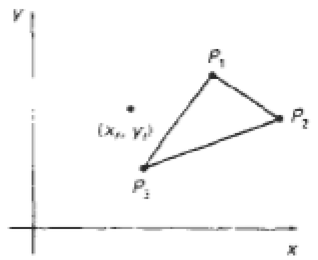

Figure 5-8
Scaling relative to a chosen fixed point (x_f, y_f). Distances from each polygon vertex to the fixed point are scaled by transformation equations 5-13.

Including coordinates for a fixed point in the scaling equations is similar to including coordinates for a pivot point in the rotation equations. We can set up a column vector whose elements are the constant terms in Eqs. 5-14, then we add this column vector to the product **S . P** in Eq. 5-12.

Polygons are scaled by applying transformations 5-14 to each vertex and then regenerating the polygon using the transformed vertices. Other objects are scaled by applying the scaling transformation equations to the parameters defining the objects. An ellipse in standard position is resized by scaling the semi-major and semi-minor axes and redrawing the ellipse about the designated centre coordinates. Uniform scaling of a circle is done by simply adjusting the radius. Then we redisplay the circle about the centre coordinates using the transformed radius.